Submitted by-

Roll No-12

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**Experiment No-12**

**Topic-** PERIODOGRAM ANALYSIS

**Problem-** The following are the temperature measurements taken every minute in a chemical reactor.

200,208,208,204,204,207,207,204,202,199,201,198,200,202,203,205,207,211,204,206,203,203,201,198,200,206,207,206,200,203,203,200,200,195,202,204

Construct the periodogram for the periods 36,18,12,9,36/5,6.Also draw up an analysis of variance table showing the mean square associated with these periods and the residual mean square.

**Theory-**

A way of analyzing a time series from which trend and seasonal variations have been eliminated is to assume that it is made up of sine and cosine waves of different frequencies such as given below.



The objective is to estimate the periodicities  (i.e. to find which are the best value of  to select) and to evaluate the constants  and . This is knownas periodogram analysis.

In periodogram analysis, we take a number of trial periods  and for each trial period , we compute the value of indicator function or intensity function. The intensity function is so choosen that when any of the trial period approaches any of true period, it attains a local maximum. The graph of the indictor function against the trial periods is known as the periodogram. From the periodogram, the trial period against which the maximum value of the indicator function occurs is taken to be estimate of the true periodicity.

Suppose the curve to be fitted to the observed time series is given by---



And n=kμ , where k is an integer and μ is a period. Then there will be k-complete rows of μ values of yt when arranged serially and extra value are not considered in the computation. The n=kμ values can be arranged in a (k×μ) arrays as follows—



This is known as Buys Ballot table. If n is not a multiple of μ (i.e. n≠kμ) we continue writing down its rows until there are fewer than μ terms left.

The constants A0,A,B are obtained by minimizing the sum of squares of errors.



With respect to A0,A and B

The least squares estimates of A0,A and B are,







The indicator function corresponding to the trial period μ is given by----



Which is also known as the indicator function of the trial period μ. I(μ) reaches a local maximum when μ tends to λ (true period). Hence it’s a plot of I(μ) versus μ , the value of μ which maximizes I(μ) say μ\* is taken to be estimate of λ.

Finally we fit the the following Fourier series model to the given time series



The ANOVA table showing the mean squares associated with the periods 36, 18, 12, 9, 36/5, 6 is given by -

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Trial period** | **Source of variation** | **df** | **SS** | **MS** | **Calculated F** | **Tabulated F** |
| 36 | 1/36 | 2 | I(36) | MS1=I(36)/2 | MS1/MSE | F(2,23) |
| 18 | 1/18 | 2 | I(18) | MS2=I(18)/2 | MS2/MSE | F(2,23) |
| 12 | 1/12 | 2 | I(12) | MS3=I(12)/2 | MS3/MSE | F(2,23) |
| 9 | 1/9 | 2 | I(9) | MS4=I(9)/2 | MS4/MSE | F(2,23) |
| 36/5 | 5/36 | 2 | I(36/5) | MS5=I(36/5)/2 | MS5/MSE | F(2,23) |
| 6 | 1/6 | 2 | I(6) | MS6=I(16)/2 | MS6/MSE | F(2,23) |
|  | **Error** | 36-1-12=23 | ESS | MSE |
| **Total** | 36-1=35 |  |  |

Where, TSS=

**Calculation-**

For calculating and evaluating the ANOVA, we use the following R-program:

y=c(200,208,208,204,204,207,207,204,202,199,201,198,200,202,203,205,207,211,204,206,203,203,201,198,200,206,207,206,200,203,203,200,200,195,202,204)

a0=mean(y)

a0

m=mat.or.vec(36,1)

n=mat.or.vec(36,1)

#36#

for(i in 1:36){

m[i]=y[i]\*cos((2\*pi\*i)/36)

n[i]=y[i]\*sin((2\*pi\*i)/36)}

a36=(2/36)\*sum(m)

a36

b36=(2/36)\*sum(n)

b36

i\_36=(36/2)\*((a36^2)+(b36^2))

i\_36

#18#

for(i in 1:36){

m[i]=y[i]\*cos((2\*pi\*i)/18)

n[i]=y[i]\*sin((2\*pi\*i)/18)}

a18=(2/36)\*sum(m)

a18

b18=(2/36)\*sum(n)

b18

i\_18=(36/2)\*((a18^2)+(b18^2))

i\_18

#12#

for(i in 1:36){

m[i]=y[i]\*cos((2\*pi\*i)/12)

n[i]=y[i]\*sin((2\*pi\*i)/12)}

a12=(2/36)\*sum(m)

a12

b12=(2/36)\*sum(n)

b12

i\_12=(36/2)\*((a12^2)+(b12^2))

i\_12

#9#

for(i in 1:36){

m[i]=y[i]\*cos((2\*pi\*i)/9)

n[i]=y[i]\*sin((2\*pi\*i)/9)}

a9=(2/36)\*sum(m)

a9

b9=(2/36)\*sum(n)

b9

i\_9=(36/2)\*((a9^2)+(b9^2))

i\_9

#36/5#

for(i in 1:36){

m[i]=y[i]\*cos((2\*pi\*i)/(36/5))

n[i]=y[i]\*sin((2\*pi\*i)/(36/5))}

a365=(2/36)\*sum(m)

a365

b365=(2/36)\*sum(n)

b365

i\_36\_5=(36/2)\*((a365^2)+(b365^2))

i\_36\_5

#6#

for(i in 1:36){

m[i]=y[i]\*cos((2\*pi\*i)/6)

n[i]=y[i]\*sin((2\*pi\*i)/6)}

a6=(2/36)\*sum(m)

a6

b6=(2/36)\*sum(n)

b6

i\_6=(36/2)\*((a6^2)+(b6^2))

i\_6

mu=c(36,18,12,9,36/5,6)

i\_mu=c(i\_36,i\_18,i\_12,i\_9,i\_36\_5,i\_6)

i\_mu

plot(mu,i\_mu,type="o",

col="blue",

xlab="Trial period ----- >",

ylab="Indicator function ----- >",

main="Periodogram",

col.main="red",

lwd=2)

grid()

tss=sum((y-a0)^2)

tss

ms=i\_mu/2

ms

ess=tss-sum(i\_mu)

ess

mse=ess/23

mse

obs\_f=ms/mse

obs\_f

tab\_f=qf(1-0.05,2,23,0)

tab\_f

The value of corresponding to the trial values of the period = 36, 18, 12, 9, 36/5, 6

|  |  |
| --- | --- |
| **Trial period** |  |
| 36 | 7.1255719 |
| 18 | 59.7049174 |
| 12 | 164.3877996 |
| 9 | 32.1076067 |
| 36/5 | 0.3472193 |
| 6 | 9.3888889 |

**ANOVA:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Trial period** | **Source of variation** | **df** | **SS** | **MS** | **Calculated F** | **Tabulated F** |
| 36 | 1/36 | 2 | 7.1255719 | 3.5627859 | 0.60391544 | 3.422132 |
| 18 | 1/18 | 2 | 59.7049174 | 29.8524587 | 5.06018638 | 3.422132 |
| 12 | 1/12 | 2 | 164.3877996 | 82.1938998 | 13.93240189 | 3.422132 |
| 9 | 1/9 | 2 | 32.1076067 | 16.0538034 | 2.72122434 | 3.422132 |
| 36/5 | 5/36 | 2 | 0.3472193 | 0.1736096 | 0.02942797 | 3.422132 |
| 6 | 1/6 | 2 | 9.3888889 | 4.6944444 | 0.79573894 | 3.422132 |
|  | **Error** | 36-1-12=23 | 135.688 | 5.899478 |
| **Total** | 36-1=35 | 408.75 |

**Conclusion-**

From the above ANOVA table we observe that F value is significant for the periods 18 and 12. Hence, we conclude that the data is a mixture of waves corresponding to the periods 18 and 12. The fitted Fourier series is



**Peridogram:**

